Electronic Raman scattering in magnetite: Spin versus charge gap

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We report Raman-scattering data of single crystals of magnetite (Fe₃O₄) with Verwey transition temperatures (T_v) of 123 and 117 K, respectively. Both single crystals reveal broad electronic background extending up to 900 wave numbers (~110 meV). Redistribution of this background is observed when samples are cooled below T_v . In particular, spectra of the low-temperature phase show diminished background below 300 wave numbers followed by an enhancement of the electronic background between 300 and 400 wave numbers. To enhance the effect of this background redistribution, we divide the spectra just below the transition by the spectra just above the transition. A resultant broad peaklike feature is observed centered at 370 ± 40 wave numbers (45 ± 5 meV). The peak position of this feature to a spin or charge gap in magnetite.

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I. INTRODUCTION

21 Magnetite (Fe_3O_4) is a naturally occurring mineral which 22 is of interest to remarkably different fields of science. It is 23 the first magnetic material known to mankind and it is the 24 earliest compound known to manifest a charge-ordering tran-25 sition discovered by Verwey¹ in 1939. Magnetite is also an 26 integral part of many living organisms. For instance, magne-27 totactic bacteria² and pigeons³ use it for navigation along the 28 Earth's magnetic field. Furthermore, it was reported that 29 magnetite occurs in human brains and may play a role in the 30 pathogenesis of the neurodegenerative diseases such as 31 Alzheimers.⁴

In condensed-matter physics, magnetite has recently attracted substantial attention^{5,6} because its charge carriers exhibit strong spin polarization at the Fermi level. This compound has the potential to become one of the leading materials for spintronics. This has initiated an interest in final-quality films of magnetite on a semiconductor substrate. Such films could form the core element of a ferromagnetsemiconductor device.⁶

40 Magnetite has been extensively studied for more than 60 41 years, yet the physics of this compound is not completely 42 understood. Competition between electronic, lattice, and 43 magnetic degrees of freedom presents a substantial challenge 44 in describing physics of magnetite. Difficulty in successful 45 modeling of this iron oxide creates a nagging reminder for 46 the scientific community as it tries to tackle such many ele-47 ment compounds as high-temperature superconductors and 48 colossal magnetoresistance compounds.

49 Verwey transition in magnetite still remains an unsolved 50 puzzle. At ambient pressure, the Verwey transition of pure or 51 near-stoichiometric magnetite is on the first order. This tran-52 sition occurs at $T_v \sim 123$ K, with changes in crystal struc-53 ture, latent heat, and a two-order-of-magnitude decrease in 54 dc conductivity. Oxygen deficiency or doping may reduce the transition temperature, may cause the transition to be- 55 come higher order, or may suppress it completely. There are 56 several competing models of the transition including Verwey 57 and Hayaaman's⁷ original order-disorder transition theory, 58 Anderson's⁸ long-range order (LRO)-short-range order 59 (SRO) model, Cullen and Callen's⁹ theory based on pure 60 electron correlations, and polaron-based theory of the 61 transition.^{10–13} However, none of these theories successfully 62 describe the whole body of experimental data. Recently 63 x-ray, neutron-, and electron-diffraction experiments have 64 cast considerable doubt on both Verwey and Hayaaman's 65 order-disorder model and Anderson's LRO-SRO model.^{14,15} 66

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Recent photoemission studies^{16,17} of magnetite indicate 67 that there is no closing of the electronic band gap at the 68 Verwey transition but rather shrinking of it by about 50 meV. 69 The latter led to the statement that the Verwey transition is 70 not actually metal to insulator transition but rather a semiconductor (or bad metal) to semiconductor transition.¹⁶ In 72 this context, questions naturally arise about the nature and 73 value of the gap in this compound. 74

Photoemission, with all its advantages, is a surface sensi-**75** tive technique.¹⁶ Depending on the incoming photon energy, **76** it probes somewhere between 15 and 45 Å. That corre-**77** sponds to not more than a dozen magnetite unit cells. In **78** contrast, the optical spectroscopy, such as Raman and infra-**79** red spectroscopy, probes to the depth of about 1000 Å. Fur-**80** thermore, the resolution of the optical methods is about one **81** order of magnitude better than that of photoemission. Table I **82** summarizes available data for the gap between the upper **83** edge of single-particle density of states and chemical poten-**84** tial. Note that this gap corresponds to the half of the "opti-**85** cal" gap obtained from the infrared data

The optical band gap detected in the infrared **87** experiment^{18,19} may have two origins. It could be the open- **88** ing of a charge gap but it can also be due to the decrease in **89** carrier mobility because of the polaron "condensation" at the **90**

TABLE I. The value of the gap obtained from the photoemission and optical measurements of magnetite.

Measurement method	Gap above the Verwey transition (meV)	Gap below the Verwey transition (meV)
Photoemission ^a	40	90
Photoemission ^b	100	150
Infrared ^c		70
Infrared ^d		100

^aReference 16.

^bReference 17.

^cReference 18.

^dReference 19.

91 Verwey transition. Unfortunately, it is difficult to separate **92** these two processes.

 Raman spectroscopy is a powerful technique capable of addressing the issue of the gap in the magnetite. Such a gap should be manifested in the electronic background of mag- netite's Raman spectrum as low intensity broad peaks. One could expect a diminished background at the frequencies smaller than the gap with subsequent increase in the scatter- ing. Similar behavior has been observed in Raman studies of HTSC compounds.²⁰ Motivated by this fact, we looked for potential signatures of the gap opening in the Raman spectra of Fe₃O₄.

103 In this paper, we present electronic Raman scattering data 104 obtained on two samples of magnetite grown by two differ-105 ent groups. We believe that we observe a "gap" feature in our 106 spectra. We discuss the origins of this gap, particularly ad-107 dressing two competing explanations. We conclude that this 108 gap feature we observe has magnetic origin.

109 II. EXPERIMENTAL DETAILS

110 In this work, we used two magnetite crystals from two 111 different sources. The first crystal was grown by Berger and 112 co-workers^{19,21} in EPFL Lausanne. This was the same crystal 113 used for our previous measurements in magnetite. This crys-114 tal was grown by a chemical vapor transport technique using 115 stoichiometric Fe₃O₄ microcrystalline powder obtained by 116 the reduction reaction of ferric oxide (Fe₂O₃). This proce-117 dure yielded near-stoichiometric single crystals with typical 118 size of $4 \times 4 \times 1$ mm³. X-ray diffraction confirmed the 119 spinel-like structure of the crystals. Transport measurements 120 detect drop of conductivity at $T_v = 123$ K.

121 The second crystal with T_V =117 K was provided by Dr. 122 Pimenov from the University of Wurzburg. This crystal was 123 originally grown by Brabers and crystals from the same 124 batch were used for the photoemission measurements of Ref. 125 16.

 Raman measurements were carried out on freshly cleaved surfaces of the as-grown single crystals. The 532 nm line of a cw solid-state laser as well as 514 and 488 nm lines of an Ar^+ ion laser were used as excitation with no more than 10 mW incident power on the sample, in order to avoid oxidation.²² Estimated overheating of the sample did not exceed 13 K. Polarized Raman spectra were measured using a 132 Dilor XY modular triple spectrometer equipped with a liquid 133 nitrogen cooled charge-coupled device (CCD) detector. The 134 spectra were measured in nearly back scattering geometry. 135 The sample temperature was maintained in He-bath cryostat 136 over a range of 4.2–300 K. 137

III. RESULTS AND DISCUSSION 138

The effects of the Verwey transition on the Raman spectra 139 of magnetite have been addressed in a number of 140 publications.^{19,21–27} Below the Verwey transition magnetite 141 has a much bigger unit cell¹⁴ that leads to a dramatic increase 142 in the number of phonon modes. In addition to the phonon 143 modes, magnetite displays a rather broad electronic back- 144 ground extending up to 900 cm⁻¹. Zero-momentum elec- 145 tronic and magnetic excitations in magnetite are likely 146 source of this background. 147

This paper focuses on the evolution of this background 148 above and below the transition. Figure 1 displays a series of 149 unpolarized spectra of magnetite near the Verwey transition. 150 One can clearly see the enhancement of the low-frequency 151 background in the high-temperature phase. This effect is ob-152 served for both samples of magnetite in Figs. 1(a) and 1(b). 153 To understand this enhancement, we need to note that the 154 Stokes Raman-scattering cross section is directly propor-155 tional to the imaginary part of the Raman response function 156 χ multiplied by the Bose factor, 157

$$\frac{d^2\sigma}{d\Omega d\omega} \propto \left\{\frac{1}{1 - e^{-\hbar\omega/k_B T}}\right\} \chi''(\omega, T).$$
(1) 158

The imaginary part of the response function contains all the 159 relevant information about quasiparticle excitations in the 160 sample under investigation. The observed background en- 161 hancement in the high-temperature phase cannot be ac- 162 counted by the Bose-factor contribution alone since Eq. (1) 163 would be virtually unchanged over the small temperature 164 shift at the Verwey transition. 165

To understand the polarization dependence of this enhancement, we measured the polarized spectra of magnetite. 167 Figures 2 and 3 display Raman data in the xx polarization. In 168 both figures, we display the Raman response function obtained by dividing corresponding spectra by the Bose factor. 170 Overheating of 13 K was taken into account. Spectrum be-171 low the Verwey transition (solid line) demonstrates redistri-172 bution of the electronic background as compared to the spec-173 trum above the transition (dotted line). There is an 174 enhancement of the background above 300 wave numbers 175 and clear depletion of the background below 400 wave numbers.

To underline the redistribution of the background, one can **178** look at the frequency range between 400 and 500 wave num- **179** bers. This spectral range has a single phonon mode at **180** 470 cm^{-1} that is absent above the Verwey transition. For this **181** frequency range, both Figs. 2 and 3 indicate that the back- **182** ground below transition is substantially smaller than that **183** above the transition. All this in spite of the fact that addi- **184** tional phonon mode appears in this spectral range below the **185** transition temperature. So even though there is added spec- **186**



FIG. 1. Unpolarized Raman spectra of magnetite in the XX geometry near Verwey transition in the (a) 123 K and (b) 117 K samples. Spectra are taken with temperature increments of 4 K. Dashed line indicates the spectra above the Verwey transition and solid line indicates the spectra below the Verwey transition. Estimated overheating was around 13 K. Note new phonon modes below the transition and strong redistribution of the electronic background as sample undergoes the transition.

187 tral weight due to the phonon mode, the background remains188 diminished compared to the above transition spectrum. We189 believe that such behavior is consistent with the opening of190 the gap below 375 wave numbers.

In order to get a better sense of the background redistri-191 192 bution, we divide the response function below the transition 193 temperature by that above the transition. The net result is a 194 peaklike feature with the maximum around 350 wave num-195 bers (insets of Figs. 2 and 3). We assign the peak position of 196 that feature to the value of the gap. Clearly, strong phonon 197 modes in the low-temperature phase do not allow for the 198 precise determination of the gap value; however we believe **199** that the manifestation of the gap opening is evident. Varying 200 the excitation line frequency of our Ar⁺ laser did not signifi-201 cantly reduce the phonon mode intensities. Our estimate of 202 the uncertainty of this gap value is 40 wave numbers 203 (\sim 5 meV). This feature is observed only in the XX polar-204 ization. Corresponding XY spectra did not display any en-205 hancement of the background.

206 One observes a number of new phonon modes in the low-207 temperature phase. It is interesting to note that the modes



FIG. 2. Polarized (XX geometry) Raman spectra of magnetite with T_v =123 K. Shown are spectra above the transition at 125 K (dashed line) and below the transition at 50 K (solid line). Inset displays the ratio of the 50 K spectrum to that above the transition at 125 K. We assign a broad peaklike feature around 350 wave numbers to the opening of a gap. Horizontal dotted line at the spectra ratio equals to one is a guide for the eyes to appreciate the redistribution of the background.

below 300 cm⁻¹ have smaller linewidth compared to the 208 modes above this frequency (Fig. 4). This could be an addi- 209 tional indication of strong interaction of the phonon modes 210 with the opening of the gap 211

The value of the gap obtained from our experiment is at **212** odds with that from the infrared data. One can make the **213** following suggestions to reconcile these differences. (i) The **214**



FIG. 3. Polarized (XX geometry) Raman spectra of magnetite with T_v =117 K. Shown are spectra above the transition at 125 K (dashed line) and below the transition at 50 K (solid line). Inset displays the ratio of the 50 K spectrum to that above the transition at 125 K. We assign a broad peaklike feature around 350 wave numbers to the opening of a gap. Horizontal dotted line at the spectra ratio equals to one is a guide for the eyes to appreciate the redistribution of the background.



FIG. 4. The linewidth of the phonon modes in the 123 K (open diamonds) and 117 K (open circles) samples as a function of their frequency as measured at 50 K (below the Verwey transition). The linewidth was obtained from the fit to the Lorentzian line shape. Dotted line is a guide for the eyes. Strong increase in the linewidth for the phonon modes above 370 cm^{-1} is evident. The increase is observed in both magnetite samples.

 infrared gap^{18,19} value may be affected by the increase in carrier mobility associated with the polaron "condensation" at the Verwey transition; (ii) the infrared gap is averaged over whole Brillouin zone (BZ) whereas Raman scattering displays a *k*-dependent charge gap. Photoemission data do indicate strong dependence of the photoemission gap on the crystallographic direction.¹⁶

222 Having made these statements, we recognize that differ-223 ences between the values of the Raman and photoemission/ir 224 gap are nearly a factor of 2 which may be too much to be 225 accounted by the potential anisotropy of the charge gap. 226 Therefore, an alternative explanation of our data is in order. 227 Such explanation comes from the recent neutron-228 scattering experiments of McQueeney *et al.*²⁸ In this work, 229 several acoustic and optical spin-wave modes (magnons) 230 were studied. Some of these magnetic excitations have been 231 originally measured in the seminal neutron-scattering experi-232 ment of Brockhouse,²⁹ however never detected by optical 233 measurements.

234 McQueeney's neutron data²⁸ indicated one particular 235 acoustic magnon that is very sensitive to the Verwey transi-236 tion. Above transition, neutron scattering detects this magnon 237 as a peak at (4,0, -1/2) reciprocal space extending from 30 238 to 50 meV (roughly 240–500 wave numbers). The magnon 239 peak itself demonstrates isotropic dispersion around the 240 (004) Brillouin-zone center.²⁸ Below the Verwey transition, 241 this peak splits into two peaks. The latter has been inter-242 preted as an opening of a gap in the acoustic magnon branch. 243 In the reciprocal space, the gap is localized at $\pm 0.1-0.2$ 244 reciprocal-lattice units²⁸ along (0.0,1) direction. This gap is 245 opening at 43 meV at the wave vector $\mathbf{q} = (0,0,\frac{1}{2})$. This 246 value of the "spin-wave gap" is in excellent agreement with 247 our data.

Raman scattering probes zone-center excitations in solids. 248 Acoustic magnon has zero energy at zero momentum and 249 therefore it cannot be detected through the conventional Ra- 250 man process. However, two-magnon scattering may alleviate 251 this restriction. The mechanism of this two-magnon scatter- 252 ing assumes the interaction of two magnons with opposite q 253 vectors. Combined excitation will therefore have zero net 254 momentum but nonzero energy. Such excitation is averaged 255 over Brillouin zone and can be probed by the appropriate 256 Raman vertex. Based on this assumption, we would expect 257 two-magnon Raman scattering to manifest itself as a broad 258 peak.^{30,31} Furthermore, when the gap develops in the magnon 259 branch we would expect a drop in the intensity of these two- 260 magnon peak beyond the energy of the gap. XX Raman ver- 261 tex probes over whole BZ. Furthermore, the location of the 262 gap in the magnon branch is consistent with this geometry. 263 Since gap develops at substantially lower temperature, we 264 can also expect narrowing of the two-magnon peak. 265

In addition to magnetic excitations, one can expect elec- 266 tronic excitations to play important role in the magnetite. In 267 general one should see electronic excitations associated with 268 carriers. The scattering rate^{32,33} is related to the slope of the 269 imaginary part of the Raman response function $\frac{\partial \chi''}{\partial \omega}$ in the 270 limit of $\omega \rightarrow 0$. The smaller the slope, the shorter is the life- 271 time τ and the larger is the scattering rate $\Gamma = 1/\tau$. Resistivity 272 is directly proportional to the scattering rate. When magne- 273 tite undergoes the Verwey transition, its resistivity increases 274 by about two orders of magnitude. The latter means that the 275 carrier scattering rate should exhibit a jump at the transition. 276 That means that the low-temperature spectrum of magnetite 277 should display low-frequency scattering characterized by 278 substantially flatter slope. That is indeed the case in our 279 experiment. 280

This "spin gap" may be associated with the optical pho- **281** non mode crossing the acoustic magnon branch. McQueeney **282** *et al.*²⁸ indicated possible flattening of the magnon branch at **283** the opening of the gap. Such flattening may create a peaklike **284** singularity in Raman data, as we observe in our data. **285**

Additional evidence for the spin gap nature of the ob- 286 served Raman effects may come from the phonon linewidths 287 displayed in Fig. 4. Below the transition in both samples, the 288 phonon modes become substantially broader above 289 $370-380 \text{ cm}^{-1}$. This is likely an indication of strong cou- 290 pling between magnetic and lattice excitations (hence, XX 291 polarization of the "spin gap" effects³⁴), providing additional 292 evidence of the spin gap. One should keep in mind that most 293 of these modes are associated with the same Fe-O tetrahe- 294 dron; therefore the increase may not be interpreted as merely 295 different groups of ions that are somehow more affected by 296 the disorder. Increase in the linewidth is likely an indicator of 297 additional decay channel for the phonons.

The XX geometry above the transition displays just three 299 phonon modes (Figs. 1 and 2). These modes are centered at 300 310, 545, and 670 cm⁻¹ (Figs. 1 and 2). The widths of the 301 modes at 150 K are 60, 29, and 63 wave numbers, respec- 302 tively. Clearly these numbers do not follow the low- 303 temperature trend displayed in Fig. 4. Only below the tran- 304 sition we observe a broadening of the modes centered above 305 370 wave numbers. This clearly dovetails the idea of the spin 306 gap opening effect on the phonon modes width. It is there- 307

 fore plausible to assign the 45 meV feature we observed in the Raman spectrum to the "spin-wave gap." This assign- ment, however reasonable, assumes that the charge gap is absent in Raman. This is probably due to small matrix ele- ments for these particular excitations. We are not aware of any theoretical work addressing the issue of Raman tensors and the charge gap in magnetite. Our result points out the need for such calculations. Further experiments with Fe ions substituted by nonmagnetic ions will shed more light on to this issue.

318 IV. CONCLUSION

319 In conclusion, we present the Raman measurements of the 320 electronic background in magnetite. Comparison of the spec-

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tra above and below the Verwey transition yields the gap 321 value of the order of 45 ± 5 meV, which is in excellent 322 agreement with recent neutron-scattering data on the spin- 323 wave gap at 43 meV. 324

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